

Introduction

'Can you do addition?' the White Queen asked. 'What's one and one and one and one and one and one and one and one and one and one and one and one?'

'I don't know,' said Alice. 'I lost count.'

'She can't do addition,' the Red Queen interrupted.

Lewis Carroll, Alice Through the Looking Glass. 1871

The power of dealing with numbers is about as common as the power of moving ears voluntarily, which is a moderately rare endowment.

Oliver Wendell Holmes. 1809-1894

The start of the lesson is important because the whole lesson tends to take its tone from the first five or ten minutes. Therefore it is vital that each mathematics lesson has a crisp start, and that the children are involved right away in some mathematics with which they can all engage.

During the first part of the lesson, the teacher almost always works directly with the whole class. The children do mental and oral work and often there will not be anything written down or recorded. This is not the time where new mathematical topics are introduced. Rather, it is the occasion when many vital mathematical skills, which benefit from, or require, small amounts of practice on a very regular basis, can be visited, rehearsed and developed.

The content may consist of a number of different activities:

- Counting in all its shapes and forms - saying or singing counting rhymes and songs; counting jumps or handclaps; counting on or back one so as to recognize 'the number after' and 'the number before'; counting in 1s, 10s, 100s, 1000s, in 50s, 500s, in 0.1s, 0.01s and 0.5s; counting in fractions such as one half or one third; counting backwards as well as forwards, starting the count from any number, not just from zero; and counting back through zero into negative numbers.
- Recalling number facts, including addition and subtraction facts, and multiplication and division facts.
- Rehearsing previously taught strategies, such as adding 9 by adding 10 and subtracting 1. Such strategies include, for example, rounding and then compensating by adding or subtracting an appropriate number, e.g. $34 + 28 = 34 + 30 - 2$, and multiplying by partitioning and adding e.g. $13 \times 5 = 10 \times 5 + 3 \times 5 = 50 + 15 = 65$.
- Developing 'new' facts, e.g. working out $26 + 7$ by using $20 + 6 + 7$, or working out six sevens by doubling three sevens, or finding $56 \div 8 = 7$ by knowing that $7 \times 8 = 56$.

Occasionally, the oral and mental starter will involve work on shapes, e.g. imagining:

- a line of square tiles, and working out how many paper tiles of the same size would be needed to surround the line completely;
- what shapes could be made if a paper square were folded in half;
- what shapes could be made if its four corners were cut off;
- what shapes could be made by overlapping two paper squares, or two square tiles.

It is important to remember that the start of the lesson:

- is usually number work;
- sometimes relates to the main activity, but sometimes does not;

is almost always with the whole class.

The approach to calculation

An ability to calculate mentally lies at the heart of numeracy. You should emphasize mental methods from the early years onwards with regular opportunities for all pupils to develop the different skills involved. These skills include:

- remembering number facts and recalling them without hesitation;
- using the facts that are known by heart to figure out new facts: for example, a fact like $8 + 6 = 14$ can be used to work out $80 + 60 = 140$, or $28 + 6 = 34$;
- understanding and using the relationships between the 'four rules' to work out answers and check results: for example, $24 \div 4 = 6$, since $6 \times 4 = 24$;
- drawing on a repertoire of mental strategies to work out calculations like $81-26$, 23×4 or 5% of £3000, with some thinking time;
- solving problems like: 'Can I buy three bags of crisps at 35p each with my £1 coin?' or: 'Roughly how long will it take me to go 50 miles at 30 m.p.h.?''

An emphasis on mental calculation does not mean that written methods are not taught in the primary years but the balance between mental and written methods, and the way in which pupils progress from one to the other, is very important.

The first stages

In the early years children will use oral methods, in general moving from counting objects or fingers one by one to more sophisticated mental counting strategies. Later they will use a number line or square to work out their answers in different ways, depending on the numbers involved. After giving them experience of a variety of situations, real and imagined, you should teach them to remember and recall simple number facts such as 5 add 3 is 8 or that 7 taken from 9 leaves 2. Posing problems and expressing relationships in different ways, and encouraging children to use this language when they talk about mathematics, is an important stage in developing their calculation strategies and problem-solving skills.

These early stages of mental calculation are not, however, at the exclusion of written recording. Alongside their oral and mental work children will learn first to read, interpret and complete statements like $5 + 8 = \square$ or $13 = \square + 5$, and then to record the results of their own mental calculations in the correct way, using a horizontal format like $43 - 8 = 35$. They should also be taught addition and subtraction alongside each other so that they are able to write the subtraction corresponding to a given addition sum, and vice versa.

The first stage of recording calculations

Pupils learn to read number statements and interpret signs and symbols. They write answers only, to develop or practise rapid recall. For example:

$$\begin{array}{ll} 6 + 4 = & \square + 10 = 17 \\ 26 + 4 = & 17 - \square = 10 \\ 36 + 4 = & \end{array}$$


Larger numbers and informal jottings

As pupils progress to working with larger numbers they will learn more sophisticated mental methods and tackle more complex problems. They will develop some of these methods intuitively and some you will teach explicitly.

Through a process of regular explanation and discussion of their own and other people's methods they will begin to acquire a repertoire of mental calculation strategies. At this stage, it can be hard for them to hold all the intermediate steps of a calculation in their heads and so informal pencil and paper notes, recording some or all of their solution, become part of a mental strategy. These personal jottings may not be easy for someone else to follow but they are an important staging post to getting the right answer and acquiring fluency in mental calculation.

The next steps in recording calculations

Pupils make jottings to assist their mental calculations:

e.g. $47 + 26$ 

Pupils record steps so that you and they can see what they have done: e.g.

$$\begin{array}{l} 36 + 27 \\ 36 + 20 \rightarrow 56 \\ 56 + 7 \rightarrow \underline{63} \\ \text{or} \\ 30 + 20 \rightarrow 50 \text{ and } 6 + 7 \rightarrow 13 \\ 50 + 13 \rightarrow \underline{63} \end{array}$$

Not everyone does a mental calculation like $81 - 26$ in the same way (nor is it necessary for them to do so) but some methods are more efficient and reliable than others. By explaining, discussing and comparing different part written, part mental methods, you can guide pupils towards choosing and using the methods which are most efficient and which can be applied generally. At this point, the need for more formal recording of calculation methods emerges.

Standard written methods

Standard written methods are reliable and efficient procedures for calculating which, once mastered, can be used in many different contexts. But they are of no use to someone who applies them inaccurately and who cannot judge whether the answer is reasonable. For each operation, at least one standard written method should be taught in the later primary years but the progression towards these methods is crucial, since they are based on steps which are done mentally and which need to be secured first. For example, the calculation of $487 + 356$, done by the method which has been taught traditionally, requires the mental calculations $7 + 6 = 13$, $8 + 5 + 1 = 14$ and $4 + 3 + 1 = 8$, while a division calculation such as $987 \div 23$ can involve mental experiment with multiples of 23 before the correct multiple is chosen.

Most countries, and in particular those which are most successful at teaching number, avoid the premature teaching of standard written methods in order not to jeopardize the development of mental calculation strategies. The bridge from recording part written, part mental methods to learning standard methods of written calculations begins only when children can add or subtract reliably any pair of two-digit numbers in their heads, usually when they are about 9 years old.

Using standard written methods

Pupils write to work out complex calculations that they cannot do mentally: e.g.

$$\begin{array}{r} 253 \\ 576 \\ + 843 \\ \hline 1672 \end{array} \qquad \begin{array}{r} \\ | 1624 \\ \hline \end{array}$$

When they have reached the stage of working out more complex calculations using pencil and paper you should still expect your pupils to practice and develop their mental calculation strategies. When faced with any calculation, no matter how large or how difficult the numbers may appear to be, the first question pupils should always ask themselves is: 'Can I do this in my head?' They then need to ask themselves: 'Do I know the approximate size of the answer?' so that they can be reasonably sure their calculation is right.

The role of calculators

The calculator is a powerful and efficient tool. It has a strong part to play in subjects such as geography, history or science, since it allows children of primary age to make use of

real data - often numbers with several digits - that they have gathered in their research or experiments, perhaps to work out a percentage, or to compare totals or proportions.

In the primary years, the calculator's main role in mathematics lessons is not as a calculating tool, since children are still developing the mental calculation skills and written methods that they will need throughout their lives. But it does offer a unique way of learning about numbers and the number system, place value, properties of numbers, and fractions and decimals. For example, you could use an overhead projector calculator for whole-class demonstration purposes so that the class can predict what happens when they multiply by 10 or divide by 10, or individual pupils might use a calculator to find two consecutive numbers with a given product and then discuss their different approaches.

If children are to use the basic facilities of a calculator constructively and efficiently, you need to teach them the technical skills they will require: the order in which to use the keys; how to enter numbers such as sums of money, measurements or fractions; how to interpret the display; how to use the memory... Children need to learn when it is, and when it is not, appropriate to use a calculator, and their first-line strategy should involve mental calculations wherever possible. For example, you might show pupils that they can 'beat the calculator' if they can recall number facts rapidly. They should also have sufficient understanding of the calculation in front of them to be able to decide which method to use - mental, pencil and paper, or calculator. When they do use a calculator they should be able to draw on well-established skills of rounding numbers and calculating mentally to gain a sense of the approximate size of the answer, and have strategies to check and repeat the calculation if they are not sure whether it is right.

For these reasons schools should not normally use the calculator as part of Key Stage 1 mathematics but should emphasize oral work and mental calculation. But by the end of Key Stage 2, pupils should have the knowledge and competence to use a calculator to work out, say, $(56 + 97) \div (133 - 85)$ and round the answer to one decimal place. They should also recognize that an approximate answer is $150 \div 50$, or 3, and use this to check their calculation.